CBCS SCHEME

USN | | | | | | | | | |

15MATDIP31

Third Semester B.E. Degree Examination, Aug./Sept.2020
Additional Mathematics – I

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Express $\frac{5+2i}{5-2i}$ in the form $xi + iy$. **(06 Marks)**

b. Find the modulus and amplitude of $\frac{(1+i)^2}{3+i}$ **(05 Marks)**

c. If $\vec{a} = (3, -1, 4)$, $\vec{b} = (1, 2, 3)$, $\vec{c} = (4, 2, -1)$ find $\vec{a} \times (\vec{b} \times \vec{c})$ **(05 Marks)**

OR

2 a. Prove that $(1+\cos\theta + i\sin\theta)^n + (1+\cos\theta - i\sin\theta)^n = 2^{n+1}\cos^n\frac{\theta}{2}\cdot\cos\frac{n\theta}{2}$. **(06 Marks)**

b. Find the sine of angle between $\vec{a} = 2i - 2j + k$ and $\vec{b} = i - 2j + 2k$ **(05 Marks)**

c. Find the value of λ, so that the vector $\vec{a} = 2i - 3j + k$, $\vec{b} = i + 2j - 3k$ and $\vec{c} = j + \lambda k$ are coplanar. **(05 Marks)**

Module-2

3 a. If $y = \tan^{-1}x$, prove that
$$(1+x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0$$ **(06 Marks)**

b. Find the angle between the radius vector and tangent to the curve $r = a(1 - \cos\theta)$ **(05 Marks)**

c. If $u = \sin^{-1}\left(\frac{x^3+y^3}{x+y}\right)$ prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\tan u$. **(05 Marks)**

OR

4 a. Find the pedal equation of the curve $r = 2(1 + \cos\theta)$ **(06 Marks)**

b. Find the total derivative of $u = x^3y^2$, where $x = e^t$, $y = \log t$. **(05 Marks)**

c. Obtain the Maclaurin's series expansion of the function sin x. **(05 Marks)**

Module-3

5 a. Evaluate $\int_0^{\pi} x\cos^6 x\,dx$. **(06 Marks)**

b. Evaluate $\int_0^1\int_0^3 x^3y^3\,dx\,dy$ **(05 Marks)**

c. Evaluate $\int_0^1\int_0^1\int_0^1 (x+y+z)\,dx\,dy\,dz$ **(05 Marks)**

OR

6 a. Evaluate $\int_0^{\pi/2} \sin^6 x \cos^5 x \, dx$ using Reduction formula. **(06 Marks)**

b. Evaluate $\int_0^1 \int_x^{\sqrt{x}} xy \, dy \, dx$ **(05 Marks)**

c. Evaluate $\int_0^1 \int_0^1 \int_0^y xyz \, dx \, dy \, dz$ **(05 Marks)**

Module-4

7 a. A particle moves along the curve $\vec{r} = (t^3 - 4t)i + (t^2 + 4t)j + (8t^2 - 3t^3)k$. Determine the velocity and acceleration at $t = 2$. **(06 Marks)**

b. Find the directional derivative of $\phi = x^2yz + 4xz^2$ at $(1, -2, -1)$ in the direction of $2i - j - 2k$. **(05 Marks)**

c. Find the constants a and b, such that $\vec{F} = (axy + z^3)i + (3x^2 - z)j + (bxz^2 - y)k$ is irrotational. **(05 Marks)**

OR

8 a. Find the angle between the tangents to the curve $x = t^2 + 1,\ y = 4t - 3,\ z = 2t^2 - 6t$ at $t = 1$ and $t = 2$. **(06 Marks)**

b. Find $\text{div}\vec{F}$ and $\text{curl}\vec{F}$ where $\vec{F} = (3x^2 - 3yz)i + (3y^2 - 3xz)j + (3z^2 - 3xy)k$ **(05 Marks)**

c. Find 'a' for which $\vec{F} = (x + 3y)i + (y - 2z)j + (x + az)k$ is solenoidal. **(05 Marks)**

Module-5

9 a. Solve $\dfrac{dy}{dx} = 1 + \dfrac{y}{x} + \left(\dfrac{y}{x}\right)^2$ **(06 Marks)**

b. Solve $x^2 \dfrac{dy}{dx} = 3x^2 - 2xy + 1$ **(05 Marks)**

c. Solve $(x^2 + y)dx + (y^3 + x)dy = 0$ **(05 Marks)**

OR

10 a. Solve $\dfrac{dy}{dx} = e^{x-y} + x^2 e^{-y}$ **(06 Marks)**

b. Solve $x \dfrac{dy}{dx} = y + x\cos^2\left(\dfrac{y}{x}\right)$ **(05 Marks)**

c. Solve $(x^4 + y^2)dy = 4x^3 y \, dx$ **(05 Marks)**

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